Math 116 Section 04

Midterm 3 Name ________________________

August 11, 2005

Instructor: Charles Cuell Student Number ________________

All solutions are to be presented on the paper in the space provided. The exam is closed book, no calculators. Time for the exam is 50 minutes.

(1) (**5 marks**) Let $u = \sqrt{x}$. Then $du = \dfrac{1}{2u}\,dx$

$$\begin{aligned}
\int \sqrt{x}\,\sin(\sqrt{x})\,dx &= \int u \sin u (2u\,du) \\
&= 2\int u^2 \sin u\,du \\
&= 2\int u^2(-\cos u)'\,du \quad \text{by parts} \\
&= 2\left(-u^2\cos u + \int 2u\cos u\,du\right) \\
&= 2\left(-u^2\cos u + 2\int u(\sin u)'\,du\right) \\
&= 2\left(-u^2\cos u + 2\left(u\sin u - \int \sin u\,du\right)\right) \\
&= 2\left(-u^2\cos u + 2u\sin u + 2\cos u\right) + C \\
&= -2x\cos\sqrt{x} + 4\sqrt{x}\sin\sqrt{x} + 4\cos\sqrt{x} + C
\end{aligned}$$

(2) (**5 marks**) Let $x = \sec\theta$. Then $dx = \sec\theta\tan\theta d\theta$, and

$$\begin{aligned}
\int \frac{dx}{x\sqrt{x^2-1}}\,dx &= \int \frac{\sec\theta\tan\theta}{\sec\theta\sqrt{\sec^2\theta-1}}\,d\theta \\
&= \int \frac{\tan\theta}{\sqrt{\tan^2\theta}}\,d\theta \\
&= \int d\theta \\
&= \theta + C \\
&= \sec^{-1} x + C
\end{aligned}$$

(3) (**5 marks**) Using polynomical division:

$$\begin{aligned}\int \frac{x^2}{x-4}\,dx &= \int \left(x+4-\frac{16}{x-4}\right)\,dx \\ &= \frac{x^2}{2}+4x-16\ln|x-4|+C\end{aligned}$$

(4) Use Simpson's Rule with $n = 4$ to estimate $\int_1^5 \frac{1}{x^2}\,dx$. $x_0 = 1, x_1 = 2, x_2 = 3, x_3 = 4, x_4 = 5$. Then $y_i = f(x_i)$ gives $y_0 = 1, y_1 = \frac{1}{4}, y_2 = \frac{1}{9}, y_3 = \frac{1}{16}, y_4 = \frac{1}{25}$. Simpson's rule gives

$$\begin{aligned}
\int_1^5 \frac{1}{x^2}\,dx &\approx \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4) \\
&= \frac{1}{3}(1 + 1 + \frac{2}{9} + \frac{4}{16} + \frac{1}{25})
\end{aligned}$$